

# Relation between CPT Violation in Neutrino masses and mixings

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## Abstract

The neutrino parameters determined from the solar neutrino data and the anti-neutrino parameters determined from KamLAND reactor experiment are in good agreement with each other. However, the best fit points of the two sets differ from each other by about  $10^{-5}$  eV<sup>2</sup> in mass-square difference and by about  $2^\circ$  in the mixing angle. Future solar neutrino and reactor anti-neutrino experiments are likely to reduce the uncertainties in these measurements. This, in turn, can lead to a signal for CPT violation in terms a non-zero difference between neutrino and anti-neutrino parameters. In this paper, we propose a CPT violating mass matrix which can give rise to the above differences in both mass-squared difference and mixing angle and study the constraints imposed by the data on the parameters of the mass matrix.

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## I. INTRODUCTION

KamLAND experiment has established the distortion due to oscillations in the anti-neutrino spectrum from reactors and determined the corresponding mass-square difference,  $\overline{\Delta}_{21}$ , to a great precision [1, 2]. At present there is good agreement between  $\overline{\Delta}_{21}$  and the mass-squared difference of the neutrinos,  $\Delta_{21}$ , determined from the analysis of solar neutrino data [3, 4]. However, the best-fit values of the two  $\Delta$ s differ from each other by about  $10^{-5}$  eV<sup>2</sup>. Also, the best-fit mixing angles differ from each other by 2 to 3 degrees. Together, solar and KamLAND data impose the constraint  $|\Delta_{21} - \overline{\Delta}_{21}| \leq 1.1 \times 10^{-4}$  eV<sup>2</sup> [5]. Future reactor experiments, located at a distance of about 70 Km from the source so that the oscillation minimum coincides with spectral maximum, are expected to improve the precision of anti-neutrino parameters even further [6]. Similarly future solar neutrino experiments [7], are expected to improve the accuracy of neutrino parameters. If these future experiments confirm the present trend in the difference between neutrino and anti-neutrino parameters, then CPT violation in the neutrino sector becomes an exciting possibility [8, 9].

When it comes to the larger mass-squared difference, the atmospheric neutrino data prefers equal values for the neutrino and anti-neutrino mass-squared differences, though the uncertainties do allow large CPT violating effects [10]. MINOS experiment is expected to measure the disappearance probability of muon anti-neutrinos with good precision in near future. If there is any difference between  $P(\nu_\mu \rightarrow \nu_\mu)$  and  $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu)$ , it will be a signal for CPT violation in the larger mass-squared difference also [11].

We assume that the main part of neutrino mass matrix is CPT conserving and it arises due to dynamics at an energy scale much below the scale at which CPT violation occurs. We further assume that CPT violation from a high scale, leads to an addition to the neutrino mass matrix of the form

$$\mathcal{M}_{\text{CPT}} = \mu \lambda_{\alpha\beta} \tag{1}$$

where  $\alpha$  and  $\beta$  are flavour indices and  $\mu$  is a parameter with dimensions of mass. As we show in the next section, to reproduce the allowed differences between neutrino and anti-neutrino parameters, we require the scale of  $\mu$  to be  $\sim 10^{-6}$  eV. With this as input, we calculate the difference in the mass-squared differences and mixing angles for the neutrinos and anti-neutrinos.

Quantum gravity effects can lead to CPT violation. The leading effective operators of

quantum gravity are suppressed as the inverse of the Planck mass  $M_{Pl}$ . Such operators can give rise to CPT violating mass  $\sim v^2/M_{Pl}$ , where  $v$  is a low energy VEV of the quantum gravity model. If  $v = 174$  GeV, the electroweak symmetry breaking scale, then we obtain  $\mu \sim 10^{-6}$  eV. The sign of the additional mass matrix for anti-neutrinos will have the opposite sign [12].

In eq. (1),  $\lambda_{\alpha\beta}$  is a  $3 \times 3$  matrix in flavour space. Quantum gravity effects are not sensitive to flavour. Hence it is expected that every term in the matrix  $\lambda_{\alpha\beta}$  is independent of both  $\alpha$  and  $\beta$ . We take this matrix to be of the form  $\lambda_{\alpha\beta} = 1$  for all  $\alpha$  and  $\beta$ . In this case, the CPT violating part of the neutrino mass matrix is of the form:

$$\mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (2)$$

In our calculations, we take eq. (2) as a perturbation to the main part of the neutrino mass matrix. The pattern of CPT violation in neutrino and anti-neutrino mass-squared difference due to the above matrix was analyzed in [13]. Here we consider the constraints from data on the CPT violating parameters in mass-squared differences and mixing angles.

## II. CALCULATION

We assume that the CPT conserving part of the light neutrino mass has real and non negative eigenvalues  $M_i$ . In the mass eigenbasis, this matrix appears as  $M = \text{diag}(M_1, M_2, M_3)$ . We treat  $M$  as the unperturbed ( $0^{th}$  - order) mass matrix. Denoting the corresponding neutrino mixing matrix by  $U$ , we obtain the  $0^{th}$  - order mass matrix  $\mathbf{M}$  in flavour space as

$$\mathbf{M} = U^* M U^\dagger. \quad (3)$$

Explicitly, the matrix  $U$  has the form

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}, \quad (4)$$

where the nine elements are functions of three mixing angles and six phases. In terms of the above elements, the mixing angles are defined by

$$\left| \frac{U_{e2}}{U_{e1}} \right| = \tan \theta_{12}, \quad (5)$$

$$\left| \frac{U_{\mu 3}}{U_{\tau 3}} \right| = \tan \theta_{23}, \quad (6)$$

$$|U_{e3}| = \sin \theta_{13}. \quad (7)$$

In terms of the above mixing angles, the MNS matrix is written as

$$U = \text{diag}(e^{if_1}, e^{if_2}, e^{if_3}) R(\theta_{23}) \Delta R(\theta_{13}) \Delta^* R(\theta_{12}) \text{diag}(e^{ia_1}, e^{ia_2}, 1). \quad (8)$$

The matrix  $\Delta = \text{diag}(e^{\frac{i\delta}{2}}, 1, e^{-\frac{i\delta}{2}})$  contains the Dirac phase  $\delta$ . This phase leads to CP violation in neutrino oscillations.  $a_1$  and  $a_2$  are the so called Majorana phases, which affect the neutrinoless double beta decay.  $f_1, f_2$  and  $f_3$  are usually absorbed as a part of the definition of the charged lepton fields. It is possible to rotate these phases away, if the mass matrix in eq. (3) is the complete mass matrix. However, since we are going to add another contribution to this mass matrix, these phases of the zeroth order mass matrix can have an impact on the complete mass matrix and thus must be retained. By the same token, the Majorana phases which are usually redundant for oscillations have a dynamical role to play now.

Given the above 0th order (CPT conserving) neutrino mass matrix, we now add the CPT violating mass matrix to it. Thus the complete light neutrino mass matrix contains both CPT conserving and CPT violating terms. Given that  $\mu$  is much smaller than the light neutrino mass scale (which should be greater than  $\sqrt{\Delta_{atm}}$  eV), we can treat the CPT violating mass matrix to be a perturbation of the CPT conserving mass matrix. The complete neutrino mass matrix in flavour space is

$$\mathbf{M} \rightarrow \mathbf{M}' = \mathbf{M} + \mu\lambda, \quad (9)$$

We assume that the symmetries inherent in  $\mathbf{M}$  lead to tribimaximal mixing. But  $\mu\lambda$  breaks these symmetries. And hence the mixing angles given by the total mass matrix  $\mathbf{M}'$  will not be tribimaximal. Below we compute the deviations from tribimaximality induced by  $\mu\lambda$  as well as the differences in mass-squared splittings. Note that these deviations will be equal and opposite for neutrinos and anti-neutrinos because  $\mu\lambda$  is CPT violating and is assumed to have opposite signs for particle and anti-particle.

The perturbation formalism, by which the above computation can be done, was first developed in ref. [14]. Here we briefly recall the main features for completeness. The matrix relevant for oscillation physics is the following hermitian matrix

$$\mathbf{M}'^\dagger \mathbf{M}' = (\mathbf{M} + \mu\lambda)^\dagger (\mathbf{M} + \mu\lambda). \quad (10)$$

To the first order in the small parameter  $\mu$ , the above matrix is

$$\mathbf{M}^\dagger \mathbf{M} + \mu \lambda^\dagger \mathbf{M} + \mathbf{M}^\dagger \mu \lambda. \quad (11)$$

This hermitian matrix is diagonalized by a new unitary matrix  $U'$ . The corresponding diagonal matrix  $M'^2$ , correct to first order in  $\mu$ , is related to the above matrix by  $U' M'^2 U'^\dagger$ . Rewriting  $\mathbf{M}$  in the above expression in terms of the diagonal matrix  $M$  we get

$$U' M'^2 U'^\dagger = U(M^2 + m^\dagger M + M m) U^\dagger, \quad (12)$$

where

$$m = \mu U^T \lambda U. \quad (13)$$

Here  $M$  and  $M'$  are the diagonal matrices with neutrino masses correct to  $0^{th}$  and  $1^{st}$  order in  $\mu$ . It is clear from eq. (12) that the new mixing matrix can be written as:

$$U' = U(1 + i\delta\Theta), \quad (14)$$

where  $\delta\Theta$  is a hermitian matrix that occurs to first order in  $\mu$ . Oscillation physics is unchanged under the transformation  $U \rightarrow UP$ , where  $P$  is a diagonal phase matrix. We can use this invariance to set the diagonal elements of the matrix  $\delta\Theta$  to be zero.

From eq. (12) we obtain

$$M^2 + m^\dagger M + M m = M'^2 + [i\delta\Theta, M'^2]. \quad (15)$$

Therefore to first order in  $\mu$ , the mass squared difference  $\Delta M_{ij}^2 = M_i^2 - M_j^2$  get modified as

$$\Delta M_{ij}'^2 = \Delta M_{ij}^2 + 2(M_i \text{Re}[m_{ii}] - M_j \text{Re}[m_{jj}]). \quad (16)$$

The non-diagonal elements of  $\delta\Theta$  are given by

$$(\delta\Theta)_{ij} = \frac{i \text{Re}(m_{ij})(M_i + M_j)}{\Delta M_{ij}'^2} - \frac{\text{Im}(m_{ij})(M_i - M_j)}{\Delta M_{ij}'^2}, \quad (17)$$

from which the changes in the mixing matrix can be computed by substituting  $\delta\Theta$  in eq. (14).

The changes induced by the small parameter  $m$  are all proportional to the neutrino mass eigenvalues. They will have their largest values in the case of degenerate masses. Hence we assume degenerate neutrino masses  $M_i \simeq M$  from hereon. In the expression for  $(\delta\Theta)_{ij}$  in

Eq. (17), the second term is utterly negligible compared to the first, if we use degenerate masses. Thus we get a greatly simplified expression [15]

$$(\delta\Theta)_{ij} = \frac{2iMRe(m_{ij})}{\Delta M_{ij}^2}, \quad (18)$$

where we have substituted the 0th order mass-square difference in the denominator because the numerator already contains a factor of  $m$ . From Eq. (18), it is trivial to see that  $(\delta\Theta)_{12}$ , whose expression contains  $\Delta_{21}$  in the denominator, is the largest among the  $(\delta\Theta)_{ij}$ .

Given the form of  $(\delta\Theta)_{ij}$ , the elements of the modified mixing matrix can be obtained as

$$U'_{\alpha j} = U_{\alpha j} + \delta U_{\alpha j} = U_{\alpha j} + i \sum_{i=1}^3 U_{\alpha i} (\delta\Theta)_{ij}. \quad (19)$$

Knowing  $U'_{\alpha j}$ , we can define the modified mixing angles  $\theta'_{ij}$  in analogy to the three equations given in Eqs. (5)-(7). To compute  $\theta'_{ij}$ , we first need to compute the changes in the five matrix elements  $\delta U_{ej}$  ( $j = 1, 2, 3$ ) and  $\delta U_{\alpha 3}$  ( $\alpha = e, \mu, \tau$ ). Given that  $(\delta\Theta)_{13}, (\delta\Theta)_{23} \ll (\delta\Theta)_{12}$ , we can easily show that the changes in  $\theta_{13}$  and  $\theta_{23}$  are very small. To obtain  $\theta'_{12}$ , we need to evaluate the [15]

$$\delta U_{e1} = -U_{e2} \frac{Re(m_{12})}{M_2 - M_1}, \quad (20)$$

$$\delta U_{e2} = U_{e1} \frac{Re(m_{12})}{M_2 - M_1}, \quad (21)$$

For later convenience we define the complex numbers  $z_i = U_{ei} + U_{\mu i} + U_{\tau i}$ , where  $U_{\alpha i}$  are, in general, functions of all six phases.

In terms of the modified mixing matrix elements,  $\theta'_{12}$  is defined as

$$\tan \theta'_{12} = \left| \frac{U'_{e2}}{U'_{e1}} \right| \quad (22)$$

Substituting the expressions from eqs. (19)-(21) in eq. (22), we get

$$\begin{aligned} \tan \theta'_{12} &= \tan \theta_{12} + 2 \frac{\mu M}{\Delta M_{21}^2} \frac{|z_1||z_2|}{\cos^2 \theta_{12}} \cos(a_1 + a_2) \cos(a_1 - a_2) \\ &= \tan \theta_{12} + \varepsilon_\theta \end{aligned} \quad (23)$$

The modified solar mass-square difference is given by

$$\begin{aligned} \Delta M_{21}'^2 &= \Delta M_{21}^2 + 2\mu M \left[ |z_2|^2 \cos(2a_2) - |z_1|^2 \cos(2a_1) \right] \\ &= \Delta M_{21}^2 + \varepsilon_\Delta \end{aligned} \quad (24)$$

Eqs. (23) and (24) give the modified mixing angle and mass-squared difference for neutrinos. The corresponding quantities for anti-neutrinos can simply be obtained by  $\mu \rightarrow -\mu$ . Thus we have

$$\begin{aligned}\overline{\Delta}M_{21}'^2 &= \overline{\Delta}M_{21}^2 - \varepsilon_\Delta = \Delta M_{21}^2 - \varepsilon_\Delta \\ \tan \overline{\theta}'_{12} &= \tan \overline{\theta}_{12} - \varepsilon_\theta = \tan \theta_{12} - \varepsilon_\theta\end{aligned}\tag{25}$$

Note that the change in the mixing angle and the change in the mass-square difference have very different dependence on the Majorana phases  $a_1$  and  $a_2$ . Therefore it will be straitforward to satisfy the experimental constraints for some combination of these two phases.

### III. RESULTS

As mentioned in the introduction, we assume that the symmetries of the 0th order neutrino mass matrix lead to tribimaximal mixing with  $\theta_{12} \simeq 35.2^\circ$ ,  $\theta_{13} = 0$  and  $\theta_{23} = 45^\circ$ . The best fit value for the reactor anti-neutrino mixing angle, from KamLAND, is  $\overline{\theta}'_{12} = 36.8^\circ$  [2]. For solar neutrinos it is  $\theta'_{12} = 32.6^\circ$  [3, 16]. Thus we see that the anti-neutrino mixing angle is  $1.6^\circ$  more than the tribimaximal prediction whereas the neutrino mixing angle is  $2.6^\circ$  below the prediction. The differences between the best fits and the tribimaximal prediction are not equal and opposite. But, within the experimental uncertainties, they can be taken to be  $2^\circ$ . It is possible that the shifts in the mixing angles for neutrinos and anti-neutrinos are not equal. To explain such shifts, we need to invoke, along with CPT violating high scale physics, contributions from CPT conserving high scale physics, such as planck scale effects [15]. The best fit value of reactor anti-neutrino mass-squared difference  $\overline{\Delta}M_{21}'^2 = 8 \times 10^{-5} \text{ eV}^2$  [2] and for solar neutrinos, the best fit value of the mass-squared difference is  $\Delta M_{12}'^2 = 6 \times 10^{-5} \text{ eV}^2$  [3, 16].

From eqs. (23), (24) and (25), we find  $\Delta M_{12}'^2 - \overline{\Delta}M_{21}'^2 = 2\varepsilon_\Delta = -2 \times 10^{-5} \text{ eV}^2$  and  $\theta'_{12} - \overline{\theta}'_{12} = 2\varepsilon_\theta = -4^\circ$ . First we explore the following question: For what values of the CPT violating parameter  $\mu$ , is there an agreement between the data and the hypothesis of CPT violating neutrino masses? In the introduction, we argued that  $\mu \sim 10^{-6} \text{ eV}$ . Here we take  $\mu = p \times 10^{-6} \text{ eV}$ , where  $p$  is a number between 1 to 10, and derive the constraints the data imposes on  $p$ . We take degenerate neutrino masses for light neutrinos  $M_i = 2 \text{ eV}$ , which is

the upper limit coming from tritium beta decay [17] and the neutrino mixing angles to be tribimaximal ones. Since  $\theta_{13} = 0$  in this case, the Dirac phase  $\delta$  can be set to zero without loss of generality. The zeroth order value of the smaller mass-square difference  $\Delta M_{21}^2$  is set to  $7 \times 10^{-5} \text{ eV}^2$ , which is the average of the neutrino and anti-neutrino mass-squared difference. The phases  $f_i$  are set to zero.

With these input values, the expressions for  $\varepsilon_\theta$  and  $\varepsilon_\Delta$  become

$$\begin{aligned} 2\varepsilon_\theta &= 0.04 p [\cos(2a_2) + \cos(2a_1)] = -4 * \pi/180 \\ 2\varepsilon_\Delta &= 8p \times 10^{-6} \left[ \frac{1}{3} \cos(2a_2) - \frac{2}{3} \cos(2a_1) \right] = 2 \times 10^{-5}. \end{aligned} \quad (26)$$

Simplifying these equations, we get the following two conditions on  $a_1$  and  $a_2$

$$\begin{aligned} [\cos(2a_2) + \cos(2a_1)] &= -1.75/p \\ [\cos(2a_2) - 2 \cos(2a_1)] &= 7.5/p. \end{aligned} \quad (27)$$

Solving these two equations and imposing the condition that  $-1 \leq \cos(2a_1), \cos(2a_2) \leq 1$ , gives us the lower limit  $p > 3$ . For  $p = 4$ , eq. (27) gives  $a_1 = -70^\circ$  and  $a_2 = 35^\circ$ . For  $p = 6$ , these values change to  $a_1 = -60^\circ$  and  $a_2 = 39^\circ$ . Note that the Majorana phases  $a_1$  and  $a_2$  should necessarily be non-zero to satisfy the two constraints in eq. (27).

#### IV. CONCLUSIONS

Both solar and reactor data are well explained by neutrino oscillations. Fit to solar data give a large region for the neutrino mass squared difference in the two flavor parameter space. The fit to reactor data however gives a very strongly constrained anti-neutrino mass squared difference. The best fits of the two mass squared differences are appreciably different from each other. Further improvement in KamLAND systematics and future solar neutrino data may further strengthen this discrepancy, thus giving a signal for CPT violation. We have demonstrated that flavour blind CPT violating neutrino masses from Planck scale physics can nicely accomodate this discrepancy, provided the Majorana phases of the neutrino mass matrix are appreciably large. This effect is crucially dependent on the neutrino mass spectrum and gives rise to observable difference between  $\Delta_{21}$  and  $\bar{\Delta}_{21}$  only for a degenerate neutrino mass spectrum with  $m_\nu \simeq 2 \text{ eV}$ , which is the largest allowed value from tritium beta decay data. The low value of the common mass implied by the WMAP bound [18]



leads to negligible difference between  $\Delta_{21}$  and  $\overline{\Delta}_{21}$ . This can however be compensated for by considering a slightly lower scale for the flavour blind CPT violating mass terms rather than the usual Planck scale.

As we discussed in section II, the difference between  $\Delta'_{31}$  and  $\overline{\Delta}'_{31}$  is negligible in this scenario if  $\mu \sim 10^{-6}$  eV. If MINOS experiment were to observe a signal for CPT violation [11], the above difference should be of order  $10^{-3}$  eV<sup>2</sup>. Accounting for such a large CPT violation in the current scenario requires the CPT violating mass parameter to be of the order of  $10^{-3}$  eV. To obtain such a large value, the scale of CPT violating physics has to be three orders below the Planck scale. The flavour matrix  $\lambda_{\alpha\beta}$ , can not be flavour blind because it would lead to an unacceptably large CPT violation for  $\Delta_{21}$ . Hence an appropriate texture should be imposed on  $\lambda_{\alpha\beta}$ .

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